УДК

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A MATHEMATICAL MODEL OF A SMALL CLASS TURBOPROP ENGINE

Results and process of a development of a mathematical model of a small turboprop engine are presented in this article. The presented model is based on engine characteristics and other necessary input data which came from the EU project ESPOSA of an engine. This engine is currently under the development and testing by present European manufacturer. The model is derived as generic, dynamic and nonlinear. There is chosen a conception, where final set of governing equations is based on a connection of submodels governing equations of elementary engine parts, i.e. compressor, turbine, nozzle. The lumped parameters method is applied to these submodels where their properties are described with characteristics, which are standard in the aircraft industry. 1st and 2nd derived mathematical models are comprised of a dynamic part which respects the rotating mass dynamics only. Result of this assumption is a system of equations, both differential and nondifferential. Dynamic part of 3th derived model respect also gas temperature and pressure dynamics. Its system of equations is fully differential.

Key words: *mathematical model, small turboprop engine, ESPOSA.*

1. Fundamentals of turboprop engine modeling philosophy

Problems in the development of mathematical models for turboprop engines are of high interest in today's control industry. There are several methods how to build a mathematical model of the arbitrary gas turbine engine in order to its control system synthesis, see [1], [2].

There were some initially failed trials of this activity at our company in history, aiming how to reach a usable result in relatively short time. Finally, an industrial standard of mathematical model design which is based on connection of submodels governing equations of basic engine parts, i.e. compressor, turbine, nozzle was accepted. The lumped parameters method is applied on these submodels where their properties are described by well-known kind of characteristics.

1.1 Model requirements

The main purpose of engine model creation is its application in the process of engine control software design and testing. Therefore, the basic set of requirements which must be met by the mathematical model of an engine is given:

– determinism – the results are dependent only on the engine inputs and have to be same in each case where the inputs are same;

– continuity in time – its desirable discretization is one of next steps;

– two-parts architecture: static model (so called engine deck) and dynamic model;

– interactivity – responds to an impulse from an engine environment;

– must run in real time;

1.2 Model assumptions

Physical nature of any industrial power plant should be correctly described by its distributed parameters. This can be realized via system of partial differential equations. This task is solved by various software based on finite volume methods (Ansys–Fluent, Comsol–Multyphysics or others) in the common industrial level. This approach is excessive and impractical in order to control algorithm design. Anyway, in order to finding of acceptable equilibrium between an accuracy and a complexity, there must be accepted following assumptions and restrictions:

– The engine = distributed physical system. Its parameters are continuously distributed in a space and time (Y=f(x,y,z,t)). Accepted assumption is a discretization of chosen control volume for particular components described by lumped parameters, see fig. 1.

– Simplification of physical properties. Thermal properties of air (dry air) and products of combustion of air with hydrocarbon fuels are described only as a function of temperature for the sake of simplicity.

– 1st and 2nd generation of his mathematical model considers only the dynamics of rotating masses. There are neglected temperature and pressure dynamics. These dynamics are considered in a 3th model generation.

– Effect of a heat accumulation inside the metal components of the engine is not considered.

– A very simple model of combustion chamber is applied. It is set as a constant of a pressure drop and combustor effectivity.

– Characteristics of the engine components are described in a form of a pressure drop and adiabatic effectivity as a functions of reduced variables. The well-known theory of similarity is applied.

– Only the adiabatic processes are considered.

1.3 Model composition

Process of the model component architecture is composed of the following steps. The first step is a decomposition of the whole power plant into the main groups and components in next step, see fig. 1. Model of the engine is assembled from components which are ordered along the engine gas path, see fig. 2

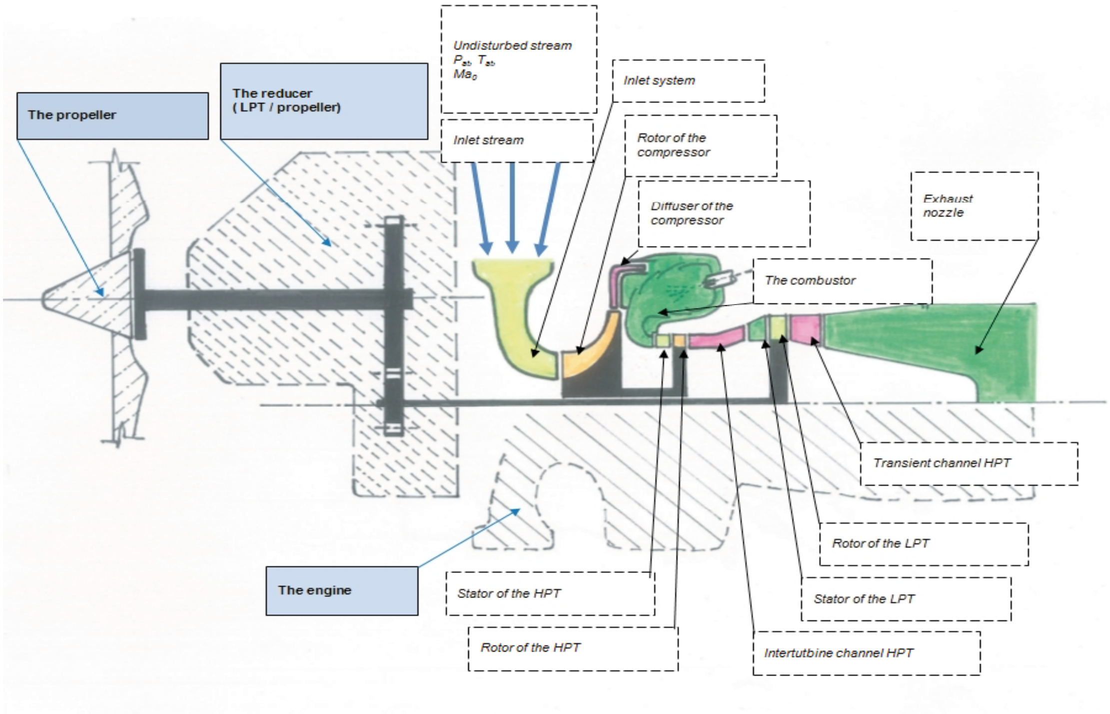

Fig. 1. Scheme of typical engine components modeled as discrete control volumes

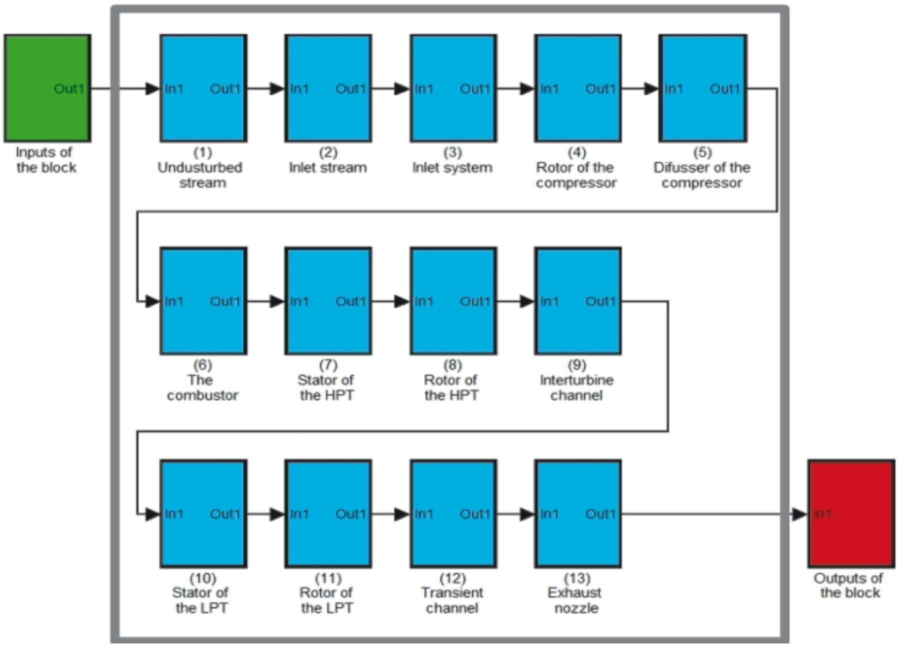

Fig. 2. Scheme of the engine gas path components

These components are joined together in the sense of serial/parallel block connections which are subsequently merged into a static and dynamic model, see fig. 3.

2. Derivation of the engine model

Borders between components in the gas path are numbered according to the aerospace standard SAE ARP755C, see fig. 4.

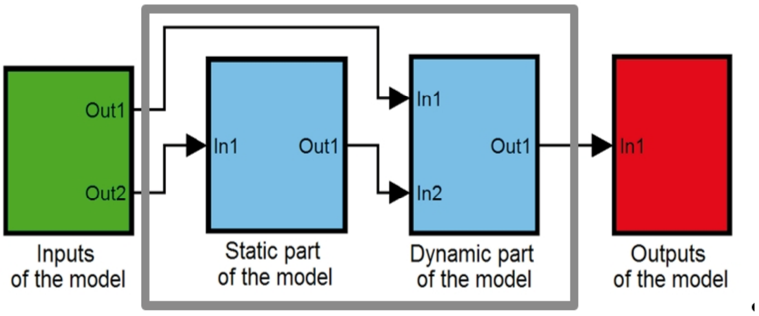

Fig.3. Scheme of the model with static and dynamic parts

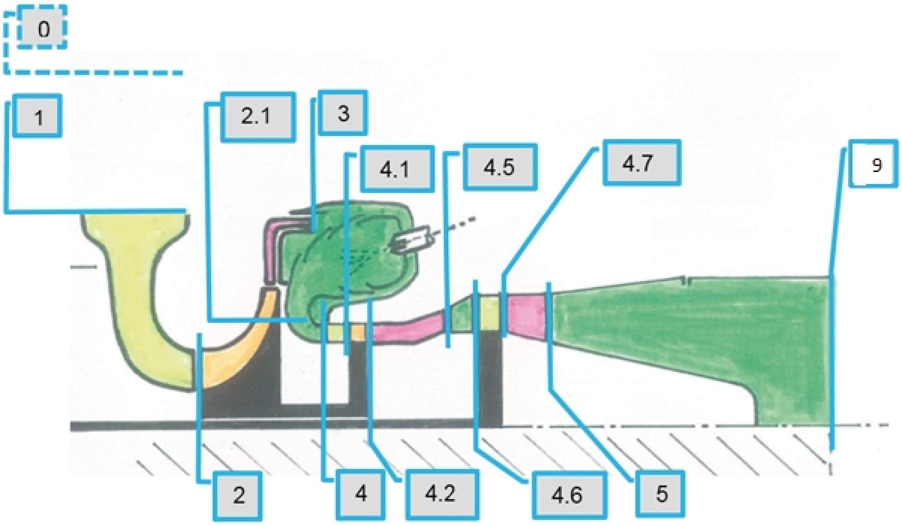

Fig. 4. Station numbering of the engine

2.1 Static model

Governing equations for temperatures, pressures and mass flows were assembled and adjusted in these numbered stations. The common laws of energy and mass conservation and various semi-empirical estimations of dimensionless values of efficiencies, pressures and speed ratios of mass flow parameters were applied.

2.2 Dynamic model

A dynamic model were assembled with the help of the static model. The governing equations were rewritten from equations of the static model, see two examples below.

1. Dynamic equilibrium of gas generator rotating mass, [rpm/s]:

$$\frac{dn_{gg}}{d\tau} = \frac{\Delta P_{gg}}{n_{gg} J_{gg} \left(\frac{\pi}{30}\right)^2}.$$

2. Dynamic equilibrium of free turbine rotating mass, [rpm/s]:

$$\frac{dn_{prop}}{d\tau} = \frac{\Delta P_{1pt}}{n_{prop} (J_{1pt} ksi_{prop} + J_{prop}) \left(\frac{\pi}{30}\right)^2}.$$

2.3 Component characteristics

Chosen block values are incorporated in the engine model. This characteristics deals with maps of pressure ratios and efficiency which are dependent on a speed ratio and mass flows parameters. These characteristics are used in calibrated forms, in the 2nd and 3rd model, see fig. 5.

3. Results of calculation of the engine model

Calculation of important values (pressures, temperatures, ...) are realized only in the numbered stations, see fig. 4.

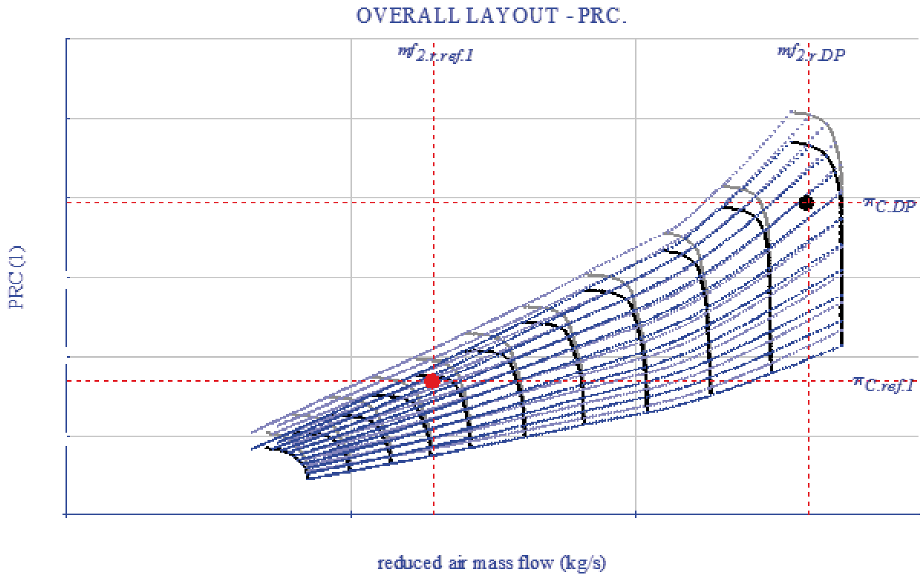

Fig. 5. Example of a calibrated compressor characteristic

3.1 Calculation of the steady state series

Calculations of above-mentioned values in the steady state are processed via the static model, so called engine deck. Examples of pressure and temperature gas path profiles are depicted in fig. 6. These values were calculated by the 1st engine model. The 2nd engine model is more precise. It enables to calculate similar set of steady state values - examples of static characteristics as a function of steady state fuel flow and Mach number, see fig. 7. Tab.1 shows calculated steady-state series of chosen values in the points of equilibrium.

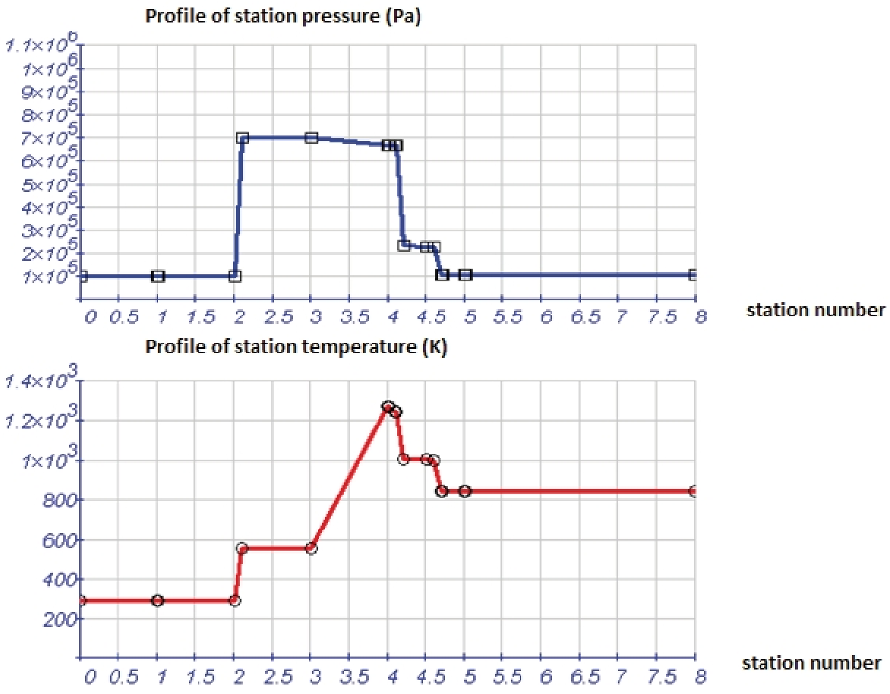

Fig. 6. Engine pressure and temperature profiles

Tab. 1

Example of a rig test at referential altitude 4000 m (ISA). The steady-state series of chosen values are calculated as a function of steady-state fuel flow

CASE	***	H.e	4000	(m)	***	delt.Te	0	(K)	***	M0.e	0	(1)	***	***	***	***
order	mf.F	mf.2	beta.C	n.GG	n.LPT	n.PP	T.3	T.4	T.41	T.42	T.44	T.5	P.3	p.42	p.5	fi.PP
(1)	(kg/s)	(kg/s)	(1)	(rpm)	(rpm)	(rpm)	(K)	(K)	(K)	(K)	(K)	(K)	(Pa)	(Pa)	(Pa)	(rad)
1	50	1,070989	0,8747896	35064,64	30829,04	2019,5899	441,45404	972,68538	947,98481	782,42996	775,28734	706,93319	278033,32	104435,89	69549,805	0,1745329
2	50,4375	1,0776499	0,8748213	35167,479	31029,826	2032,7432	442,44971	974,81891	950,06746	783,66451	776,51291	707,35987	280087,38	104880,07	69562,623	0,1745329
3	50,875	1,0843967	0,8748293	35270,925	31240,338	2046,5338	443,44976	976,89891	952,09909	784,84248	777,68294	707,66911	282158,49	105319,8	69574,907	0,1745329
4	51,3125	1,0910916	0,8748595	35373,323	31450,274	2060,2865	444,43802	978,97646	954,12787	786,02927	778,86147	707,97991	284218,95	105758,84	69586,584	0,1745329
5	51,75	1,0974817	0,8749942	35471,576	31639,868	2072,7067	445,3855	981,14403	956,24095	787,34308	780,1643	708,51758	286217,73	106204,05	69597,546	0,1745329
6	52,1875	1,1035525	0,875235	35565,616	31807,077	2083,6605	446,29123	983,40605	958,44254	788,78917	781,59658	709,30176	288151,82	106656,17	69607,826	0,1745329
7	52,625	1,1093987	0,8755487	35656,732	31958,369	2093,5715	447,16668	985,72751	960,69965	790,32233	783,11403	710,2548	290040,17	107113,04	69617,496	0,1745329
8	53,0625	1,1151253	0,8759006	35746,375	32101,46	2102,9453	448,02437	988,07021	962,97631	791,89321	784,6683	711,28668	291903,93	107572,11	69626,617	0,1745329
9	53,5	1,1207841	0,8762746	35835,366	32240,241	2112,0368	448,87066	990,41574	965,25517	793,47768	786,23576	712,35253	293753,56	108032,17	69635,219	0,1745329
10	53,9375	1,1264043	0,8766628	35924,244	32376,891	2120,9886	449,70929	992,75402	967,52677	795,0623	787,80324	713,42723	295594,96	108492,57	69643,318	0,1745329
11	54,375	1,1320069	0,8770597	36013,454	32512,917	2129,8996	450,54299	995,07799	969,78447	796,63752	789,36139	714,49309	297432,42	108952,92	69650,924	0,1745329
12	54,8125	1,1376062	0,8774626	36103,375	32649,295	2138,8336	451,37372	997,38307	972,02399	798,19695	790,90397	715,53846	299268,87	109413,01	69658,043	0,1745329
13	55,25	1,1432044	0,8778725	36194,252	32785,989	2147,7884	452,20212	999,6691	974,24519	799,73983	792,4302	716,56291	301104,8	109872,96	69664,68	0,1745329
14	55,6875	1,1495797	0,8780559	36296,412	32819,75	2150	453,11407	1001,659	976,18678	800,91243	793,59357	716,852	303098,1	110314,5	69671,019	0,1767501
15	56,125	1,1561732	0,8781816	36401,782	32819,75	2150	454,0461	1003,5535	978,03807	801,97227	794,64636	716,92616	305135,51	110751,39	69676,892	0,1796699
16	56,5625	1,1627308	0,8783181	36505,755	32819,75	2150	454,9684	1005,4423	979,88345	803,03574	795,7026	717,02028	307165,23	111189,42	69682,237	0,1825615
17	57	1,1692559	0,8784564	36607,223	32819,75	2150	455,8798	1007,3228	981,72044	804,10159	796,76107	717,13007	309187,73	111627,98	69687,055	0,185424
18	57,4375	1,175746	0,8785909	36705,247	32819,75	2150	456,77878	1009,1949	983,54881	805,17114	797,82307	717,25446	311202,3	112066,4	69691,347	0,1882547
19	57,875	1,1821895	0,8787211	36799,072	32819,75	2150	457,66326	1011,062	985,37177	806,24982	798,89391	717,39876	313206,4	112504,29	69695,111	0,1910458
20	58,3125	1,1885778	0,8788524	36888,634	32819,75	2150	458,53229	1012,927	987,19206	807,34133	799,97724	717,56701	315198,22	112941,59	69698,35	0,1937924
21	58,75	1,1949113	0,8789997	36974,799	32819,75	2150	459,38676	1014,7904	989,01023	808,44522	801,0726	717,75903	317177,91	113378,62	69701,069	0,1964966
22	59,1875	1,2011901	0,8791759	37058,318	32819,75	2150	460,2275	1016,653	990,82693	809,56123	802,17971	717,97478	319145,57	113815,74	69703,274	0,1991609
23	59,625	1,2074129	0,8793917	37139,753	32819,75	2150	461,05511	1018,5157	992,64315	810,68971	803,29892	718,21492	321100,98	114253,26	69704,97	0,2017875
24	60,0625	1,2135773	0,8796563	37219,545	32819,75	2150	461,86999	1020,3797	994,46018	811,83151	804,43106	718,48062	323043,72	114691,47	69706,162	0,204378
25	60,5	1,2196802	0,8799782	37298,048	32819,75	2150	462,67245	1022,2468	996,27957	812,98789	805,57738	718,77352	324973,17	115130,66	69706,855	0,2069333
26	60,9375	1,2257175	0,880365	37375,558	32819,75	2150	463,46269	1024,1188	998,10304	814,16039	806,73936	719,09561	326888,58	115571,12	69707,055	0,2094539
27	61,375	1,2316879	0,8808143	37452,289	32819,75	2150	464,24142	1025,9967	999,93173	815,34948	807,91749	719,44737	328789,71	116012,79	69706,769	0,2119406
28	61,8125	1,2375863	0,8812973	37528,182	32819,75	2150	465,0098	1027,8836	1001,7686	816,55719	809,11377	719,8304	330675,71	116454,96	69706,004	0,2143947
29	62,25	1,2434093	0,881783	37603,175	32819,75	2150	465,76902	1029,782	1003,616	817,78488	810,32955	720,24547	332545,99	116896,86	69704,767	0,2168177
30	62,6875	1,2491567	0,882243	37677,271	32819,75	2150	466,52033	1031,6928	1005,475	819,03249	811,56481	720,69184	334400,63	117337,88	69703,067	0,2192116
31	63,125	1,2548306	0,8826517	37750,514	32819,75	2150	467,26501	1033,6162	1007,3457	820,29896	812,8185	721,16766	336240,18	117777,45	69700,91	0,221579
32	63,5625	1,2604352	0,8829854	37822,985	32819,75	2150	468,00435	1035,5515	1009,2275	821,58244	814,08885	721,67022	338065,51	118215,06	69698,304	0,2239229
33	64	1,2659756	0,8832219	37894,783	32819,75	2150	468,73964	1037,4975	1011,1194	822,88063	815,3736	722,1963	339877,77	118650,26	69695,254	0,2262466
34	64,4375	1,2714578	0,8833397	37966,021	32819,75	2150	469,47213	1039,4527	1013,02	824,19094	816,67022	722,74234	341678,24	119082,61	69691,766	0,2285535
35	64,875	1,2768886	0,8833296	38036,813	32819,75	2150	470,20261	1041,415	1014,9273	825,51048	817,97588	723,3047	343468,27	119511,96	69687,842	0,2308469
36	65,3125	1,2822732	0,883213	38107,234	32819,75	2150	470,93048	1043,3817	1016,8386	826,83714	819,2885	723,88121	345248,79	119938,92	69683,485	0,2331283
37	65,75	1,2876146	0,8830153	38177,322	32819,75	2150	471,65485	1045,3508	1018,7522	828,16965	820,60682	724,47078	347020,27	120364,18	69678,697	0,2353983
38	66,1875	1,2929148	0,8827613	38247,095	32819,75	2150	472,37483	1047,3209	1020,6664	829,5073	821,93012	725,0729	348782,96	120788,41	69673,482	0,2376576
39	66,625	1,2981744	0,8824753	38316,548	32819,75	2150	473,0895	1049,2908	1022,5802	830,84987	823,25818	725,68764	350536,88	121212,23	69667,841	0,2399059
40	67,0625	1,3033928	0,8821818	38385,657	32819,75	2150	473,79792	1051,2601	1024,4932	832,19768	824,5913	726,31565	352281,78	121636,23	69661,779	0,2421412
41	67,5	1,3085682	0,881905	38454,379	32819,75	2150	474,49909	1053,2287	1026,4052	833,5516	825,9303	726,95821	354017,17	122061	69655,298	0,2443589
42	67,9375	1,3136973	0,8816694	38522,65	32819,75	2150	475,19197	1055,1971	1028,3167	834,91301	827,27655	727,61726	355742,28	122487,12	69648,404	0,2465544
43	68,375	1,3187749	0,8814965	38590,381	32819,75	2150	475,87554	1057,1668	1030,2289	836,2842	828,63227	728,29568	357455,95	122915,07	69641,105	0,2487227

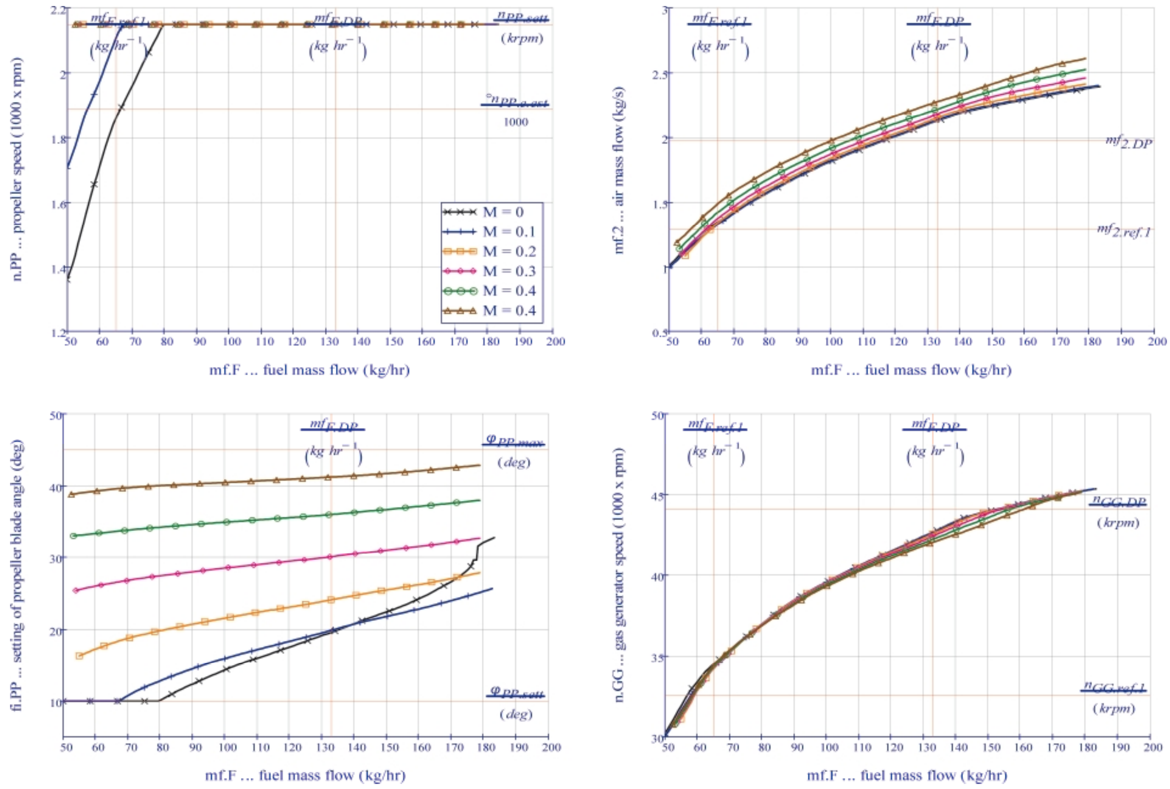

Fig. 7. Values of engine gas generator speed (nGG), compressor air mass flow (mf2), ‑propeller speed (nPP) and propeller blade angle (fiPP) as the function of steady-state fuel flow and Mach number

3.2 Calculation of the transient state series

Transient states of important values were calculated for many cases of input values combinations. These calculations were processed using MathCAD software as well as MATLAB/Simulink, see fig. 8. The MathCAD calculations are more convenient for the algorithm debugging. The MATLAB/Simulink model is advantageous for large and real-time data processing.

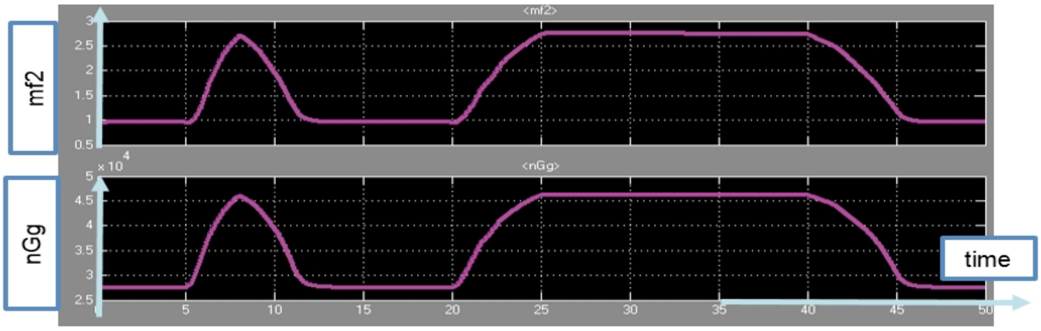

Fig. 8. Example of time series of compressor air mass flow (mf2) and gas generator speed (nGG) as a reaction to fuel mass

There was made an attempt to model development in a GSP software (Gas turbine Simulation Program) in order to get some kind of spectacular verification. An approximate example of this calculation is placed in fig. 9. This attempt for verification failed due to unclarity of input and other parameters setting.

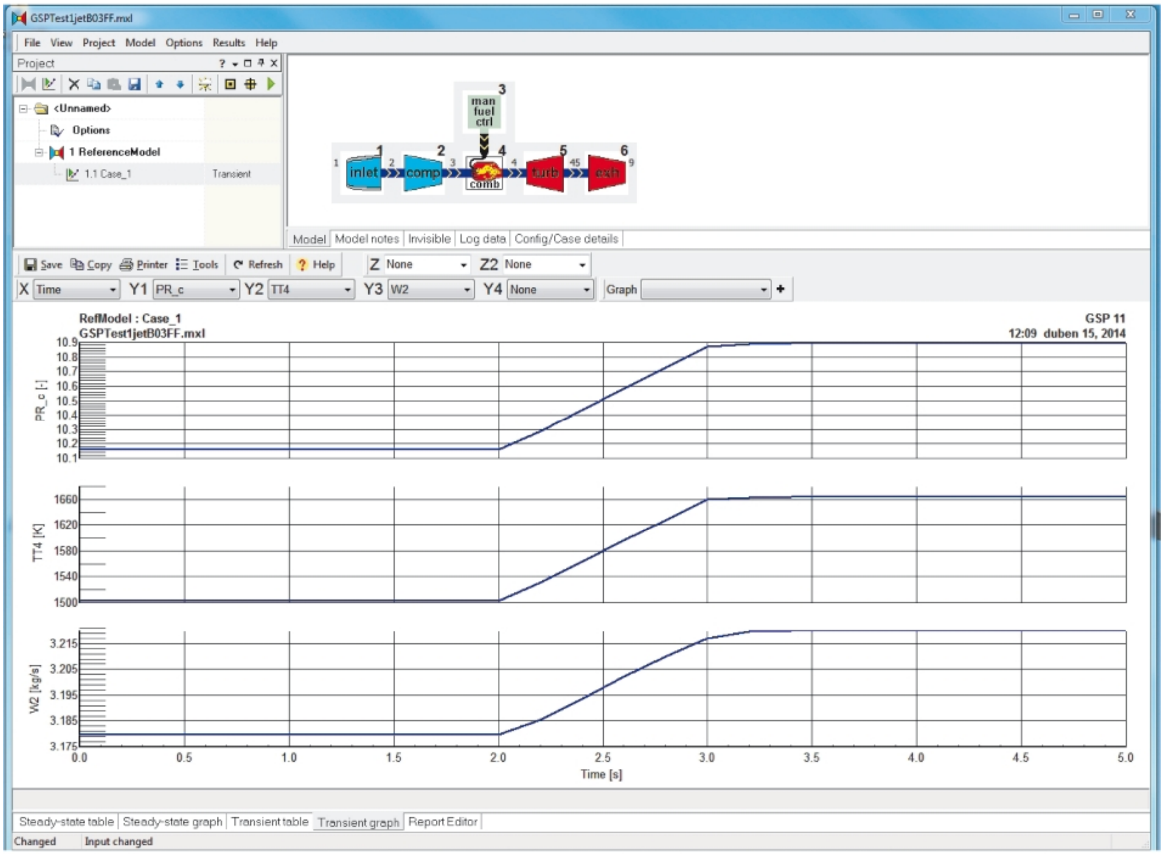

Fig. 9. Jet engine example of a GSP results sheet. There is calculated time series of compressor pressure ratio (PRC), temperature after combustor (T4) and compressor mass flow (W2)

4. Possibilities for the next development

Development of the 3^{rd} revision of the engine model successfully started in the first quarter of 2015. This approach was chosen in response to the problems with an implementation of previous two models into the dSPACE hardware-in-the-loop simulation platform. The 3^{rd} generation of model is based on fully differential set of governing equations which is the main difference in comparison with the previous model generations. Previous models were constituted of mixed set of both differential and nondifferential governing equations. This mixed system contained algebraic loops as a source of problems. The major expectations for the 3^{rd} model generation are connected with a significant shortening of calculation time in order to fulfilling of real-time requirements. Unfortunately, the lower accuracy in calculated results is expected as a penalty of additional simplifications. This drawback has been accepted in order to fast processing of the 3^{rd} engine model algorithm.

Development of the 3^{rd} engine model algorithm is being finalized nowadays in MathCAD software. In order to a future progress, there is a need to start the transition and testing of the model in MATLAB/Simulink.

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Jozef Polacek, Lubos Vargovcik. Математическая модель турбовинтового малоразмерного двигателя

В данной статье представлены результаты и описывается процесс разработки математической модели турбовинтового малоразмерного двигателя. Представленная модель основана на характеристиках двигателя и других необходимых входных данных, полученных по проекту Европейского Союза ESPOSA. Данный двигатель в настоящее время находится на стадии разработки и испытаний европейским изготовителем. Полученная модель является типовой, динамической и нелинейной. Используется концепция, согласно которой окончательный набор определяющих уравнений основан на связи определяющих уравнений субмоделей элементарных узлов двигателя, например, компрессора, турбины, соплового аппарата. Для этих субмоделей используется метод сосредоточенных параметров, где их свойства описываются характеристиками, которые являются стандартными в авиации. Первая и вторая производные математические модели состоят из динамической части, которая распространяется только на динамику вращающихся масс. Результатом этого ограничения является система как дифференциальных, так и недифференциальных уравнений. Динамическая часть третьей производной модели также распространяется на динамику температуры и давления. Ее система уравнений является полностью дифференциальной.

Ключевые слова: *математическая модель, малые турбовинтовые двигатели, ESPOSA*

Jozef Polacek, Lubos Vargovcik. Математична модель турбогвинтового малорозмірного двигуна

У даній статті представлено результати і описується процес розробки математичної моделі турбогвинтового малорозмірного двигуна. Представлена модель заснована на характеристиках двигуна і інших необхідних вхідних даних, отриманих за проектом Європейського Союзу ESPOSA. Даний двигун у цей час перебуває в стадії розробки і випробувань європейським виготовлювачем. Отримана модель є типовою, динамічною і нелінійною. Використовується концепція, відповідно до якої остаточний набір визначальних рівнянь засновано на зв'язку визначальних рівнянь субмоделей елементарних вузлів двигуна, наприклад, компресора, турбіни, соплового апарата. Для цих субмоделей використовується метод зосереджених параметрів, де їх властивості описуються характеристиками, які є стандартними в авіації. Перша і друга похідної математичної моделі складаються з динамічної частини, що поширюється тільки на динаміку обертових мас. Результатом цього обмеження є система як диференціальних, так і недиференціальних рівнянь. Динамічна частина третьої похідної моделі також поширюється на динаміку температури і тиску. Її система рівнянь є повністю диференціальною.

Ключові слова: *математична модель, малорозмірний турбогвинтовий двигун, ESPOSA.*